Overview of the SME: Implications and Phenomenology of Lorentz Violation

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Abstract

The Standard Model Extension (SME) provides the most general observer-independent field theoretical framework for investigations of Lorentz violation. The SME lagrangian by definition contains all Lorentz-violating interaction terms that can be written as observer scalars and that involve particle fields in the Standard Model and gravitational fields in a generalized theory of gravity. This includes all possible terms that could arise from a process of spontaneous Lorentz violation in the context of a more fundamental theory, as well as terms that explicitly break Lorentz symmetry. An overview of the SME is presented, including its motivations and construction. Some of the theoretical issues arising in the case of spontaneous Lorentz violation are discussed, including the question of what happens to the Nambu-Goldstone modes when Lorentz symmetry is spontaneously violated and whether a Higgs mechanism can occur. A minimal version of the SME in flat Minkowski spacetime that maintains gauge invariance and power-counting renormalizability is used to search for leading-order signals of Lorentz violation. Recent Lorentz tests in QED systems are examined, including experiments with photons, particle and atomic experiments, proposed experiments in space, and experiments with a spin-polarized torsion pendulum.

1 Introduction

It has been 100 years since Einstein published his first papers on special relativity [1]. This theory is based on the principle of Lorentz invariance, that the laws of physics and the speed of light are the same in all inertial frames. A few years after Einstein's initial work, Minkowski showed that a new spacetime geometry emerges from special relativity. In this context, Lorentz symmetry is an exact spacetime symmetry that maintains the form of the Minkowski metric in different Cartesian-coordinate frames.

In the years 1907-1915, Einstein developed the general theory of relativity as a new theory of gravity. In general relativity, spacetime is described in

terms of a metric that is a solution of Einstein's equations. The geometry is Riemannian, and the physics is invariant under general coordinate transformations. Lorentz symmetry, on the other hand, becomes a local symmetry. At each point on the spacetime manifold, local coordinate frames can be found in which the metric becomes the Minkowski metric. However, the choice of the local frame is not unique, and local Lorentz transformations provide the link between physically equivalent local frames.

The Standard Model (SM) of particle physics is a fully relativistic theory. The SM in Minkowski spacetime is invariant under global Lorentz transformations, whereas in a Riemannian spacetime the particle interactions must remain invariant under both general coordinate transformations and local Lorentz transformations. Particle fields are also invariant under gauge transformations. Exact symmetry under local gauge transformations leads to the existence of massless gauge fields, such as the photon. However, spontaneous breaking of local gauge symmetry in the electroweak theory involves the Higgs mechanism, in which the gauge fields can acquire a mass.

Classical gravitational interactions can be described in a form analogous to gauge theory by using a vierbein formalism [2]. This also permits a straightforward treatment of fermions in curved spacetimes. Covariant derivatives of tensors in the local Lorentz frame involve introducing the spin connection. In a Riemann spacetime with zero torsion, the spin connection is not an independendent field, but rather is a prescribed function of the vierbein and its derivatives. However, a natural generalization is to treat the spin connection components as independent degrees of freedom. The resulting geometry is a Riemann-Cartan spacetime, which has nonvanishing torsion [3]. In a Riemann-Cartan spacetime, the associated field strengths for the vierbein and spin connection are the curvature and torsion tensors. The usual Riemann spacetime of general relativity is recovered in the zero-torsion limit. Similarly, if the curvature tensor vanishes, the spacetime reduces to Minkowski spacetime.

The combination of the SM and Einstein's classical gravitational theory provides a highly successful description of nature. However, since Einstein's theory is not a quantum theory, it is expected that it will ultimately be superseded by a more fundamental theory that will hold at the quantum level. Candidate quantum gravity theories include string theory and loop quantum gravity. The appropriate scale where gravity and quantum physics are expected to meet up is the Planck scale, $m_P \simeq 10^{19}$ GeV.

Finding experimental confirmation of a quantum theory of gravity by doing experiments at the Planck scale, however, is not practical. Instead, an alternative approach can be adopted in which one looks for small Planck-suppressed effects of new physics that might be observable in high-precision experiments. For this idea to hold, any new effect would have to be one that cannot be mimicked by known conventional processes in the SM or conventional gravity theory. One possible signal fulfilling this requirement

is to look for Planck-suppressed signatures of Lorentz violation in high-precision experiments.

Detection of such a violation of relativity theory would clearly be a dramatic indication of new physics, presumably coming from the Planck scale. This idea is not merely speculative because it has been shown that mechanisms in both string theory [4, 5] and quantum gravity [6] can lead to violations of Lorentz symmetry. However, these theories are not yet sufficiently developed in such a way that allows testable predictions to be made at a definite (quantifiable) scale at low energies.

Nonetheless, progress can still be made using effective field theory. To be realistic, an effective field theory would have to contain both the SM and general relativity together with any higher-order couplings between them. It should also maintain coordinate (or observer) independence. In full generality, the gravity sector could include additional fields such as torsion that are not a part of Einstein's general relativity. This would permit more general geometries, including a Riemann-Cartan spacetime.

The general effective field theory of this type incorporating arbitrary observer-independent Lorentz violation is called the Standard-Model Extension (SME) [7, 8, 9]. The SME lagrangian by definition contains all observer-scalar terms consisting of products of SM and general gravitational fields with each other as well as with additional couplings that introduce violations of Lorentz symmetry. In principle, there are an infinity of terms in the SME, including nonrenormalizable terms of arbitrarily high dimension.

To investigate low-energy experiments, where the leading-order signals of Lorentz violation are of primary interest, it is often advantageous to work with a subset of the full SME, which includes only a finite number of terms. One subset in particular, referred to as the minimal SME, restricts the theory to power-counting renormalizable and gauge-invariant terms. In recent years, the Lorentz-violating coefficients in the minimal SME have been adopted by experimentalists as the standard for reporting bounds on Lorentz violation. Since many of the low-energy experiments involve only electromagnetic interactions between charged particles and photons, it often suffices to define a minimal QED sector of the SME.

This paper is intended as an overview in the context of the SME of some recent theoretical and phenomenological investigations of Lorentz violation. The motivations for the development of the SME are presented first. An outline of how the theory is constructed is then given. This is followed by a discussion of some theoretical issues that come up when Lorentz violation is due to a process of spontaneous symmetry breaking. In particular, the fate of the Nambu-Goldstone modes is examined along with the question of whether a Higgs mechanism can occur [10]. For simplicity, this discussion is carried out in the context of a vector model known as a bumblebee model [11, 9]. The role of the geometry (Minkowski, Riemann, or Riemann-Cartan) is examined as well. To investigate phenomenology, the minimal SME is

constructed and used to examine a wide range of experiments assuming a flat Minkowski background. In this paper, the focus is on high-precision tests in QED systems. A number of recent experiments in atomic and particle systems are examined, and the status of their attainable sensitivities to Lorentz violation is reviewed.

The SME is the result of a large on-going collaboration by a group of theorists and experimentalists most of whom have in common that they have at some point collaborated with Alan Kostelecky at Indiana University. An exhaustive review covering all of this collective work, which spans topics in field theory, gravity, astrophysics, cosmology, as well as particle, nuclear, and atomic physics, is not possible here. Instead, this review focuses mostly on selective recent topics that are of interest to the author. It is also not possible here to give a complete list of references on all of the work looking at possible violations or tests of relativity. For that, other recent reviews and proceedings collections should be consulted as well. See, for example, Refs. [12, 13, 14].

2 Motivations

Historically, interest in Lorentz violation increased dramatically after it was discovered by Kostelecky and Samuel in the late 1980s that mechanisms can occur in string field theory that could cause spontaneous breaking of Lorentz symmetry [4]. It is this idea that ultimately led to the development of the SME, which in turn has stimulated a variety of experimental searches for relativity violations.

Spontaneous Lorentz violation can occur when a string field theory has a nonperturbative vacuum that can lead to tensor-valued fields acquiring nonzero vacuum expectation values (vevs), $\langle T \rangle \neq 0$. As a result of this, the low-energy effective theory contains an unlimited number of terms of the form

$$\mathcal{L} \sim \frac{\lambda}{m_P^k} \langle T \rangle \, \Gamma \, \bar{\psi} (i\partial)^k \chi \quad , \tag{1}$$

where k is an integer power, λ is a coupling constant, and ψ and χ are fermion fields. In this expression, the tensor vev $\langle T \rangle$ carries spacetime indices, which are suppressed in this notation. This vev is effectively a set of functions or constants that are fixed in a given observer frame. What this means is that interactions with these coefficients can have preferred directions in spacetime or velocity (boost) dependence. The vev coefficients therefore induce Lorentz violation.

Note that the higher-dimensional ($k > 0$) derivative couplings are expected to be balanced by additional inverse factors of a large mass scale, which is assumed to be the Planck mass m_P. In a more complete low-energy effective theory describing fermions ψ and χ there could also be other terms

with additional couplings, including possible Yukawa couplings. A more general interaction term of the form in Eq. (1) at order k could then be written as

$$\mathcal{L} \sim t^{(k)}\,\Gamma\,\bar{\psi}(i\partial)^k\chi \quad , \tag{2}$$

where the coefficient $t^{(k)}$, which carries spacetime indices, absorbs all of the couplings, inverse mass factors, and the vev. This effective coefficient acts essentially as a fixed background field that induces Lorentz violation. In addition to interactions with fermions, additional terms involving gauge-field couplings and gravitational interactions are possible as well. A generalization would be to include all possible contractions of known SM and gravitational fields with fixed background coefficients $t^{(k)}$.

This generalization to include all arbitrary-dimension interaction terms inducing Lorentz violation in effective field theory is the idea behind the SME [7, 8, 9]. Note as well that each term is assumed to be an observer scalar, with all spacetime indices contracted. The full SME is then defined as the effective field theory obtained when all such scalar terms are formed using SM and gravitational fields contracted with coefficients that induce Lorentz violation. The SME coefficients (the generalized $t^{(k)}$ factors) are assumed to be heavily suppressed, presumably by inverse powers of the Planck mass. The extent of the suppression increases with order k. Without a completely viable string field theory, it is not possible to assign definite numerical values to these coefficients, and clearly (as in the SM itself) there are hierarchy issues. However, since no Lorentz violation has been observed in nature, it must be that the SME coefficients are small. Alternatively, one can adopt a phenomenological approach and treat the coefficients as quantities to be bounded in experiments. Such bounds will also constitute a measure of the sensitivity to Lorentz violation attained in the experiment.

Interestingly, although the SME was originally motivated from ideas in string field theory, including the idea of spontaneous Lorentz symmetry breaking, its relevance and usefulness extend well beyond these ideas. In fact, there is nothing in the SME that requires that the Lorentz-violation coefficients emerge due to a process of spontaneous Lorentz violation. The SME coefficients can also be viewed as due to explicit Lorentz violation or as arising from some unknown mechanism. Indeed, once the philosophy of the SME is appreciated – that it is the most general observer-independent field theory incorporating Lorentz violation – then no matter what scalar lagrangian is written down involving known low-energy fields, the result will be contained in the full SME.

An illustration of this comes from studying noncommutative field theory. These are theories that have noncommuting coordinates

$$[x^\mu, x^\nu] = i\theta^{\mu\nu} \quad . \tag{3}$$

It has been shown that this type of geometry can occur naturally in string theory [15], and that it leads to Lorentz violation [16]. Here, however, the

mechanism leading to Lorentz violation is in general different from that of spontaneous symmetry breaking. Nonetheless, the form of the effective interactions that arise are fully contained in the SME. The fixed parameters $\theta^{\mu\nu}$, which break the Lorentz symmetry, act effectively so as to produce SME coefficients. For example, the effective field theory involving a $U(1)$ gauge field in a noncommutative geometry includes lagrangian terms of the form

$$\mathcal{L} \sim \frac{1}{4} \, iq \, \theta^{\alpha\beta} \, F_{\alpha\beta} \, \bar{\psi} \, \gamma^{\mu} \, D_{\mu} \, \psi \quad , \tag{4}$$

where $F_{\alpha\beta}$ is the field strength. Here, as in Eq. (1) the interaction takes the form of a scalar-valued product of known particle fields, derivative operators, and a set of fixed background functions.

There are a number of other examples of effective field theories with Lorentz violation that have been put forward in recent years, with a wide variety of motivations or ideas for symmetry breaking. Nonetheless, as long as the resulting theories are described by scalar lagrangians, then they are compatible with the approach of the SME. For example, a model with spatial rotational invariance was used in Ref. [17] to study high-energy cosmic rays above the GZK cutoff. Another example with a higher-dimensional lagrangian giving rise to Lorentz-violating dispersion relations was considered in Ref. [18]. An example involving gravitational fields includes a parameterized set of kinetic terms for a vector field in a theory with spontaneous Lorentz breaking [19]. In all of these cases, the lagrangian terms can be found as a subset of the full SME.

Over the years, a number of phenomenological frameworks that involve specific types of Lorentz violation have been developed and used extensively by experimentalists. A sampling includes the $TH\epsilon\mu$ model [20], the Robertson-Mansouri-Sexl framework [21], the PPN formalism [22], as well as models based on kinematical breaking of Lorentz symmetry (see Refs. [12, 14] for reviews). In some cases, these theories describe parameterized equations of motion or dispersion relations and do not originate from a scalar lagrangian. However, to the extent that these models can be described by effective field theory defined by a scalar lagrangian, they are compatible with the SME and direct links between their parameterizations and the SME coefficients can be obtained.

It should be noted as well, that in addition to breaking Lorentz symmetry, the SME also leads to violation of the discrete symmetry CPT [4, 5]. This symmetry is the product of charge conjugation (C), parity (P), and time reversal (T). According to the CPT theorem [23], a relativistic field theory describing point particles should exactly obey CPT symmetry. Conversely, a second theorem states that if CPT is violated in field theory, then Lorentz symmetry must also be broken [24]. It then follows that any observer-independent effective field theory describing CPT violation must also be contained within the SME. Since CPT can be tested to very high

precision in experiments with matter and antimatter, this opens up a whole new avenue for exploring the phenomenology of Lorentz violation.

In summary, the full SME is defined as the most general observer-independent theory of Lorentz and CPT violation that contains the SM and gravity. It thus provides a unifying framework that can be used to investigate possible signals of Lorentz and CPT violation. Because it contains an infinity of terms, with an unlimited set of coefficients with spacetime indices, it is not possible to list all of them. However, the terms can be classified in a general way, and a uniform notation can be developed. It is also possible to develop subset theories of the full SME, which contain a finite number of terms. One subset in particular has been investigated extensively in recent experiments. It is the minimal SME, which is comprised of the gauge-invariant subset of terms in the full SME with dimension four or less.

Finally, one other remark about the SME coefficients is worth mentioning. It is often commented these coefficients, such as for example a nonzero vacuum vev of a tensor field generated from a process of spontaneous Lorentz violation, are reminiscent of the old pre-relativistic ether. However, the ether was thought to be a medium (with a rest frame) for light, whereas an SME coefficient need not be thought of in this way. The SME coefficients act effectively as background vacuum fields. Their interactions typically select out a particular particle species. In fact, if that particle is not the photon, then the SME coefficient will have no direct influence on the speed of light. Moreover, the SME coefficients carry tensor indices and therefore have definite spacetime directions in any observer frame. In the end, while there are some similarities to the old ether, the physical effects of the SME coefficients are significantly different.

3 Constructing the SME

One of the defining features of the SME is that the theory is observer independent [8]. It is therefore important to make clear the distinction between what are called *observer* and *particle* Lorentz transformations. An observer Lorentz transformation is a change of observer frame. It can be viewed as a rotation or boost of the basis vectors in the local frame. The philosophy of the SME is that even with Lorentz violation, physics must remain observer independent. The results of an experiment should not depend on the chosen perspective of any observer. In contrast, a particle Lorentz transformation is a rotation or boost performed on an individual particle field while leaving the coordinate frame fixed. In this case, if there is Lorentz violation, the physics can change.

In terms of what this means for an experiment, the observer invariance of the SME says that the results of a measurement cannot depend on the choice of coordinate frame or observational perspective made by the experi-

menter. On the other hand, if Lorentz symmetry is broken, the results of the experiment can change if the apparatus itself is rotated or boosted in some direction, both of which are examples of particle Lorentz transformations.

Note that this feature of the SME breaks the relativity principle, which is a central assumption of (unbroken) relativity theory. This principle is often stated as the equivalence of passive and active Lorentz transformations when one is performed as the inverse of the other. In the formulation of the SME, however, the terms passive and active are deliberately avoided since for one thing their usage is sometimes confused in the literature. More importantly, though, it is observer independence that is the physically defining feature of the theory, and the terminology should reflect this. In addition, observers need not be inactive. The idea in the SME is that even if an observer actively changes its perspective or relative motion with respect to the apparatus in an experiment, the results of measurements should remain unchanged.

A similar distinction between observer and particle transformations can be made for general coordinate transformations performed in the spacetime manifold of a Riemann or Riemann-Cartan geometry [9, 10]. An observer transformation is simply a change of spacetime coordinates, which leaves the physics unchanged. On the other hand, a particle transformation is essentially a diffeomorphism, which maps one point on the spacetime to another. The change in a tensor under pullback to the original point is given by the Lie derivative.

The full SME is defined using a vierbein formalism. This permits a natural distinction between the spacetime manifold and local Lorentz frames. The vierbein ${e_\mu}^a$ provides a link between the components of a tensor field $T_{\lambda\mu\nu\cdots}$ on the spacetime manifold (denoted using Greek indices) and the corresponding components $T_{abc\cdots}$ in a local Lorentz frame (denoted using latin indices). The correspondence is given by

$$T_{\lambda\mu\nu\cdots} = {e_\lambda}^a {e_\mu}^b {e_\nu}^c \cdots T_{abc\cdots}. \tag{5}$$

In this notation, the components of the spacetime metric are $g_{\mu\nu}$, while in a local Lorentz frame, the metric takes the Minkowski form η_{ab}. A necessary condition for the vierbein is therefore that $g_{\mu\nu} = {e_\mu}^a {e_\nu}^b \eta_{ab}$. Covariant derivatives acting on tensor fields with local indices introduce the spin connection ${\omega_\mu}^{ab}$. For example,

$$D_\mu {e_\nu}^a = \partial_\mu {e_\nu}^a - {\Gamma^\alpha}_{\mu\nu} {e_\alpha}^a + {\omega_\mu}^a{}_b {e_\nu}^b. \tag{6}$$

In a Riemann spacetime where $D_\lambda g_{\mu\nu} = 0$, the spin connection is not an independent field, but rather is a prescribed function of the vierbein and its derivatives. However, in a Riemann-Cartan spacetime the spin connection represents independent degrees of freedom associated with there being nonzero torsion.

The observer independence of the SME requires that all of the terms in the lagrangian be observer scalars under both general coordinate transformations and local Lorentz transformations. This means that every spacetime index and every local Lorentz index must be fully contracted in the lagrangian.

However, the SME is not invariant under particle diffeomorphisms and local Lorentz transformations. Explicitly, a diffeomorphism maps one point on the spacetime to another. It can be characterized infinitesimally in a coordinate basis by the transformation

$$x^\mu \to x^\mu + \xi^\mu. \tag{7}$$

The four infinitesimal parameters ξ^μ comprise the diffeomorphism degrees of freedom. On the other hand, under an infinitesimal particle Lorentz transformation the field components transform through contraction with a matrix of the form

$$\Lambda^a{}_b \approx \delta^a{}_b + \epsilon^a{}_b, \tag{8}$$

where $\epsilon_{ab} = -\epsilon_{ba}$ are the infinitesimal parameters carrying the six Lorentz degrees of freedom and generating the local Lorentz group. Evidently, there are a total of ten relevant spacetime symmetries.

Violation of these symmetries occurs when an interaction term contains coefficients that remain fixed under a particle transformation, such as when as a particle rotation or boost is performed in a background with a fixed vev.

3.1 Minimal SME

The full SME consists of an unlimited number of observer scalar terms consisting of contractions of SM fields, gravitational fields, and SME coefficients. To begin to explore phenomenology, it makes sense to advance incrementally. Since gauge symmetry and renormalizability are foundations of our current understanding in particle physics, a first increment would be to construct a subset theory that maintains these features. It is referred to as the minimal SME. It will first be defined in Minkowski spacetime and then generalized to include gravitational fields in a Riemann-Cartan geometry.

The minimal SME, constructed from dimension four or fewer operators, describes the leading-order effects of Lorentz violation. This is because the higher-dimensional terms are expected to be suppressed by additional inverse powers of the Planck mass compared to those in the minimal SME. Effects involving couplings to gravitational fields are also expected to be smaller than those involving interactions in the SM, particularly electrodynamic interactions. For this reason, the Lorentz tests described later on are investigated using primarily a QED subset of the minimal SME in flat Minkowski spacetime. Nonetheless, it should be kept in mind that a particular type of Lorentz violation might only occur at subleading order. For

this reason, it is important ultimately to investigate more general tests in the context of the full SME, including gravitational effects as well as interactions involving higher-dimensional terms. However, that goes beyond the scope of this overview.

To construct the minimal SME in flat Minkowski spacetime [8], the first ingredient that must be put in is the minimal SM itself. This consists of quark and lepton sectors, gauge fields, and a Higgs sector. Denote the left- and right-handed lepton and quark multiplets by

$$L_A = \begin{pmatrix} \nu_A \\ l_A \end{pmatrix}_L \quad , \quad R_A = (l_A)_R \quad , \tag{9}$$

$$Q_A = \begin{pmatrix} u_A \\ d_A \end{pmatrix}_L \quad , \quad U_A = (u_A)_R \quad , \quad D_A = (d_A)_R \quad , \tag{10}$$

where $A = 1, 2, 3$ labels the flavor, with $l_A = (e, \mu, \tau)$, $\nu_A = (\nu_e, \nu_\mu, \nu_\tau)$, $u_A = (u, c, t)$, and $d_A = (d, s, b)$. The Higgs doublet is denoted by ϕ. The SU(3), SU(2), and U(1) gauge fields are G_μ, W_μ, and B_μ, respectively, with corresponding field strengths: $G_{\mu\nu}$, $W_{\mu\nu}$, and $B_{\mu\nu}$. The gauge couplings are g_3, g, and g', while q denotes the electric charge. The Yukawa couplings are G_L, G_U, G_D.

The relevant sectors in the SM lagrangian are:

$$\mathcal{L}_{\text{lepton}} = \tfrac{1}{2} i \overline{L}_A \gamma^\mu \overleftrightarrow{D}_\mu L_A + \tfrac{1}{2} i \overline{R}_A \gamma^\mu \overleftrightarrow{D}_\mu R_A \quad , \tag{11}$$

$$\mathcal{L}_{\text{quark}} = \tfrac{1}{2} i \overline{Q}_A \gamma^\mu \overleftrightarrow{D}_\mu Q_A + \tfrac{1}{2} i \overline{U}_A \gamma^\mu \overleftrightarrow{D}_\mu U_A + \tfrac{1}{2} i \overline{D}_A \gamma^\mu \overleftrightarrow{D}_\mu D_A \quad , \tag{12}$$

$$\mathcal{L}_{\text{Yukawa}} = -\Big[(G_L)_{AB} \overline{L}_A \phi R_B + (G_U)_{AB} \overline{Q}_A \phi^c U_B + (G_D)_{AB} \overline{Q}_A \phi D_B \Big], \tag{13}$$

$$\mathcal{L}_{\text{Higgs}} = (D_\mu \phi)^\dagger D^\mu \phi + \mu^2 \phi^\dagger \phi - \frac{\lambda}{3!} (\phi^\dagger \phi)^2 \quad , \tag{14}$$

$$\mathcal{L}_{\text{gauge}} = -\tfrac{1}{2} \mathrm{Tr}(G_{\mu\nu} G^{\mu\nu}) - \tfrac{1}{2} \mathrm{Tr}(W_{\mu\nu} W^{\mu\nu}) - \tfrac{1}{4} B_{\mu\nu} B^{\mu\nu}, \tag{15}$$

where D_μ are gauge-covariant derivatives.

The SME introduces additional lagrangian terms that are contractions of these SM fields with the SME coefficients . The SME coefficients are constrained by the requirement that the lagrangian be hermitian. For an SME coefficient with an even number of spacetime indices, the pure trace component is irrelevant because it maintains Lorentz invariance. Such coefficients may therefore be taken as traceless.

In the fermion sector of the minimal SME, four sets of terms can be classified according to whether they involve leptons or quarks and whether CPT is even or odd. They are

$$\mathcal{L}^{\text{CPT-even}}_{\text{lepton}} = \tfrac{1}{2} i (c_L)_{\mu\nu AB} \overline{L}_A \gamma^\mu \overleftrightarrow{D}^\nu L_B + \tfrac{1}{2} i (c_R)_{\mu\nu AB} \overline{R}_A \gamma^\mu \overleftrightarrow{D}^\nu R_B \, , \tag{16}$$

$$\mathcal{L}^{\mathrm{CPT-odd}}_{\mathrm{lepton}} = -(a_L)_{\mu AB}\overline{L}_A\gamma^\mu L_B - (a_R)_{\mu AB}\overline{R}_A\gamma^\mu R_B \quad , \tag{17}$$

$$\begin{aligned}\mathcal{L}^{\mathrm{CPT-even}}_{\mathrm{quark}} &= \tfrac{1}{2}i(c_Q)_{\mu\nu AB}\overline{Q}_A\gamma^\mu \overleftrightarrow{D^\nu} Q_B + \tfrac{1}{2}i(c_U)_{\mu\nu AB}\overline{U}_A\gamma^\mu \overleftrightarrow{D^\nu} U_B \\ &\quad + \tfrac{1}{2}i(c_D)_{\mu\nu AB}\overline{D}_A\gamma^\mu \overleftrightarrow{D^\nu} D_B \quad ,\end{aligned} \tag{18}$$

$$\begin{aligned}\mathcal{L}^{\mathrm{CPT-odd}}_{\mathrm{quark}} &= -(a_Q)_{\mu AB}\overline{Q}_A\gamma^\mu Q_B - (a_U)_{\mu AB}\overline{U}_A\gamma^\mu U_B \\ &\quad -(a_D)_{\mu AB}\overline{D}_A\gamma^\mu D_B \quad .\end{aligned} \tag{19}$$

In these expressions, the coefficients a_μ have dimensions of mass, while $c_{\mu\nu}$ are dimensionless and traceless.

The couplings between fermions and the Higgs field are all CPT even and are

$$\begin{aligned}\mathcal{L}^{\mathrm{CPT-even}}_{\mathrm{Yukawa}} &= -\tfrac{1}{2}\Big[(H_L)_{\mu\nu AB}\overline{L}_A\phi\sigma^{\mu\nu}R_B + (H_U)_{\mu\nu AB}\overline{Q}_A\phi^c\sigma^{\mu\nu}U_B \\ &\quad +(H_D)_{\mu\nu AB}\overline{Q}_A\phi\sigma^{\mu\nu}D_B\Big] \quad ,\end{aligned} \tag{20}$$

where the SME coefficients $H_{\mu\nu}$ are dimensionless and antisymmetric.

The Higgs sector itself can be CPT even or odd. The terms are

$$\begin{aligned}\mathcal{L}^{\mathrm{CPT-even}}_{\mathrm{Higgs}} &= \tfrac{1}{2}(k_{\phi\phi})^{\mu\nu}(D_\mu\phi)^\dagger D_\nu\phi - \tfrac{1}{2}(k_{\phi B})^{\mu\nu}\phi^\dagger\phi B_{\mu\nu} \\ &\quad -\tfrac{1}{2}(k_{\phi W})^{\mu\nu}\phi^\dagger W_{\mu\nu}\phi \quad ,\end{aligned} \tag{21}$$

$$\mathcal{L}^{\mathrm{CPT-odd}}_{\mathrm{Higgs}} = i(k_\phi)^\mu\phi^\dagger D_\mu\phi \quad . \tag{22}$$

The dimensionless coefficient $k_{\phi\phi}$ can have symmetric real and antisymmetric imaginary parts. The other coefficients have dimensions of mass.

The gauge sector consists of

$$\begin{aligned}\mathcal{L}^{\mathrm{CPT-even}}_{\mathrm{gauge}} &= -\tfrac{1}{2}(k_G)_{\kappa\lambda\mu\nu}\mathrm{Tr}(G^{\kappa\lambda}G^{\mu\nu}) - \tfrac{1}{2}(k_W)_{\kappa\lambda\mu\nu}\mathrm{Tr}(W^{\kappa\lambda}W^{\mu\nu}) \\ &\quad -\tfrac{1}{4}(k_B)_{\kappa\lambda\mu\nu}B^{\kappa\lambda}B^{\mu\nu} \quad ,\end{aligned} \tag{23}$$

$$\begin{aligned}\mathcal{L}^{\mathrm{CPT-odd}}_{\mathrm{gauge}} &= (k_3)_\kappa\epsilon^{\kappa\lambda\mu\nu}\mathrm{Tr}(G_\lambda G_{\mu\nu} + \tfrac{2}{3}ig_3G_\lambda G_\mu G_\nu) \\ &\quad +(k_2)_\kappa\epsilon^{\kappa\lambda\mu\nu}\mathrm{Tr}(W_\lambda W_{\mu\nu} + \tfrac{2}{3}igW_\lambda W_\mu W_\nu) \\ &\quad +(k_1)_\kappa\epsilon^{\kappa\lambda\mu\nu}B_\lambda B_{\mu\nu} + (k_0)_\kappa B^\kappa \quad .\end{aligned} \tag{24}$$

The coefficients $k_{G,W,B}$ are dimensionless, have the symmetries of the Riemann tensor, and have a vanishing double trace. The coefficients $k_{1,2,3}$ are real and have dimensions of mass, while k_0 is also real and has dimensions of mass cubed. Note that if any of these CPT-odd terms appear in the theory, they would generate instabilities associated with negative contributions to the energy. For this reason, the coefficients $k_{0,1,2,3}$ are assumed to vanish.

Interestingly, it appears that no radiative corrections in the SME appear to generate nonzero values for these coefficients, at least to one loop.

It is also important to realize that some of the SME terms can be eliminated by field redefinitions [8, 25, 26]. For example, some of the terms involving the coefficients $a_{L,R,Q,U,D}$ can be eliminated by position-dependent field-phase redefinitions. Another example is that certain terms involving the coefficients $c_{L,R,Q,U,D}$ can be absorbed by the terms involving the coefficients $H_{L,U,D}$ through field-normalization redefinitions. In particular, what this means is that while a field theory can be written down that ostensibly has explicit Lorentz violation, it is sometimes the case that there are no physical effects because the theory is equivalent through field redefinitions to a Lorentz-invariant theory.

Clearly, there are a number of additional theoretical issues that are relevant to the construction of the SME as a consistent low-energy field theory incorporating Lorentz violation. These include a more in-depth discussion of the nature of field theory with Lorentz violation (including quantization of the theory) [8], issues related to causality [25], the possibility of additional extensions including for example supersymmetry [27], renormalization [28], electroweak symmetry breaking [8], radiative corrections [29], spacetime variations of couplings [30], etc. It is not possible to describe all of these issues here. The interested reader is referred to the original papers.

3.2 Gravity Sector

The gravity sector of the SME has been discussed in Ref. [9], and the minimal theory (dimension four or fewer terms) has been explicitly constructed. A vierbein formalism is used, which gives the theory a close parallel to gauge theory. Lorentz breaking occurs due to the presence of SME coefficients, which remain fixed under particle Lorentz transformations in a local frame. In this case, the SME coefficients carry Latin indices, e.g., b_a for a vector, with respect to the local basis set. The conversion to spacetime coordinates is implemented by the vierbein, giving, e.g., $b_\mu = e_\mu{}^a b_a$. The lagrangian can then be written in terms of fields and SME coefficients defined on the spacetime manifold. A natural (though not required) assumption is that the SME coefficients are smooth functions over the manifold. It is not necessary to require that they be covariantly constant. In fact, defining covariantly constant tensors over a manifold places stringent topological constraints on the geometry. One simplifying assumption, which could occur naturally in the context of spontaneous Lorentz breaking, is to assume that the SME coefficients are constants in the local frame. However, again, this is not a requirement in the formulation of the SME theory.

To construct the minimal SME including gravity, the first step is to incorporate gravitational fields into the usual SM. This is done by rewriting all of the terms in Eqs. (11) through (15) with fields and gamma matrices

defined with respect to the local frame (using Latin indices). The vierbein is then used to convert these terms over to the spacetime manifold. Factors of the determinant of the vierbein e are included as well so that integration of the lagrangian density (giving the action) is covariant. Derivatives are understood as well to be both spacetime and gauge covariant. With these changes, Eq. (11), for example, becomes

$$\mathcal{L}_{\text{lepton}} = \tfrac{1}{2} iee^{\mu}{}_{a} \overline{L}_A \gamma^a \overleftrightarrow{D_\mu} L_A + \tfrac{1}{2} iee^{\mu}{}_{a} \overline{R}_A \gamma^a \overleftrightarrow{D_\mu} R_A. \tag{25}$$

The other terms for the quark, Yukawa, Higgs, and gauge sectors follow a similar pattern.

The Lorentz-violating SME terms constructed from SM fields are obtained in a similar way. The various particle sectors can again be divided between CPT odd and even contributions. Each of the terms in Eqs. (16) to (24) is then written using local indices and vierbeins, which convert the equations over to the spacetime manifold. As an example, Eq. (16) becomes

$$\begin{aligned} \mathcal{L}^{\text{CPT-even}}_{\text{lepton}} &= -\tfrac{1}{2} i (c_L)_{\mu\nu AB} ee^{\mu}{}_{a} \overline{L}_A \gamma^a \overleftrightarrow{D^\nu} L_B \\ &\quad -\tfrac{1}{2} i (c_R)_{\mu\nu AB} ee^{\mu}{}_{a} \overline{R}_A \gamma^a \overleftrightarrow{D^\nu} R_B. \end{aligned} \tag{26}$$

The remaining equations follow the same pattern.

The pure-gravity sector of the minimal SME consists of a Lorentz-invariant gravity sector and a Lorentz-violating sector. The Lorentz-invariant lagrangian consists of terms that are products of the gravitational fields. In the general case, this includes terms constructed from curvature, torsion, and covariant derivatives. Einstein's gravity (with or without a cosmological term) would be a special case in this sector.

The Lorentz-violating lagrangian terms in the gravity sector of the minimal SME are constructed by combining the SME coefficients with gravitational field operators to produce an observer scalar under local Lorentz transformations and general coordinate transformations. These consist of products of the vierbein, the spin connection, and their derivatives, but for simplicity they can be written in terms of the curvature, torsion, and covariant derivatives. The minimal case (up to dimension four) has the form:

$$\begin{aligned} \mathcal{L}^{\text{LV}}_{e,\omega} &= e (k_T)^{\lambda\mu\nu} T_{\lambda\mu\nu} + e (k_R)^{\kappa\lambda\mu\nu} R_{\kappa\lambda\mu\nu} + e (k_{TT})^{\alpha\beta\gamma\lambda\mu\nu} T_{\alpha\beta\gamma} T_{\lambda\mu\nu} \\ &\quad + e (k_{DT})^{\kappa\lambda\mu\nu} D_\kappa T_{\lambda\mu\nu}. \end{aligned} \tag{27}$$

The SME coefficients in this expression have the symmetries of the associated Lorentz-violating operators. All except $(k_T)^{\lambda\mu\nu}$, which has dimensions of mass, are dimensionless.

The Lorentz-violating sector introduces additional gravitational couplings that can have phenomenological consequences, including effects on cosmology, black holes, gravitational radiation, and post-Newtonian physics. As a

starting point for a phenomenological investigation of the gravitational consequences of Lorentz violation, it is useful to write down the Riemannian limit of the minimal SME gravity sector. It is given as [9]

$$\begin{aligned} S_{e,\omega,\Lambda} &= \frac{1}{2\kappa}\int d^4x[e(1-u)R - 2e\Lambda \\ &\qquad + es^{\mu\nu}R_{\mu\nu} + et^{\kappa\lambda\mu\nu}R_{\kappa\lambda\mu\nu}]. \end{aligned} \tag{28}$$

The SME coefficient $(k_R)^{\kappa\lambda\mu\nu}$ has been expanded into coefficients $s^{\mu\nu}$, $t^{\kappa\lambda\mu\nu}$, u that distinguish the effects involving the Riemann, Ricci, and scalar curvatures. The coefficients $s^{\mu\nu}$ have the symmetries of the Ricci tensor, while $t^{\kappa\lambda\mu\nu}$ has those of the Riemann tensor. Taking tracelessness conditions into account, there are 19 independent components.

Another useful limit is the QED subset of the SME. This extension in Minkowski space has been used extensively to investigate high-precision experimental tests of Lorentz symmetry in atomic and particle systems. Generalizing to include gravity involves introducing additional vierbein-fermion couplings as well as a pure-gravity sector. These additional terms can then be investigated for potential signals of Lorentz violation due to gravitational effects in high-precision experiments.

A full treatment of the gravity sector of the SME should include looking at the energy momentum tensor, Einstein's equations, and consistency relations between these stemming from, for example, the Bianchi identities. These types of issues are described in depth in Ref. [9]. Interestingly, a difference between theories with explicit versus spontaneous breaking of Lorentz symmetry is found. In a generic Riemann-Cartan theory with explicit breaking of Lorentz symmetry, the Bianchi identities are not consistent with the the covariant conservation laws and equations of motion. On the other hand, if Lorentz symmetry is spontaneously broken, the problem is evaded.

4 Spontaneous Lorentz Violation

One of the original motivations for developing the SME was that mechanisms in string theory suggest that local Lorentz symmetry might be spontaneously broken [4]. While the full SME describes any observer-independent Lorentz violation at the level of effective field theory, one important special case is when Lorentz symmetry is spontaneously broken. This provides an elegant mechanism in which the symmetry holds dynamically, but is broken (or hidden) by the solutions of the theory. The lagrangian and equations of motion still respect the symmetry, however; the vacuum values of the fields do not. Tensor-valued fields acquire nonzero vevs which have definite spacetime directions, thereby breaking the symmetry under boosts and rotations.

There are certain theoretical issues that arise when the Lorentz violation is due to spontaneous symmetry breaking. This section examines some of these issues, in particular, what the fate is of the Nambu-Goldstone modes when Lorentz symmetry is spontaneously broken.

In gauge theory, it is well known that when a continuous global symmetry is spontaneously broken, massless Nambu-Goldstone (NG) modes appear [31]. If instead the broken symmetry is local, then a Higgs mechanism can occur in which the gauge bosons become massive [32]. The question naturally arises as to what the fate of the NG modes is when Lorentz symmetry is spontaneously broken and whether a Higgs mechanism can occur for the case of local Lorentz symmetry (as in a theory with gravity).

This question has recently been addressed in detail in Ref. [10]. A generic analysis of theories with spontaneous Lorentz breaking was carried out in Riemann-Cartan spacetime and in the limiting cases of Riemann and Minkowski spacetime. A number of general features were found.

First, a connection between spontaneous breaking of local Lorentz symmetry and diffeomorphisms was found to hold. This occurs because when the vierbein takes a vacuum value, which for simplicity we can take as its value in a Minkowski background, ${e_\mu}^a = \delta^a_\mu$, then if a local tensor acquires a fixed vev, e.g., b_a for the case of a vector, which breaks local Lorentz symmetry, then the associated spacetime vector b_μ as given by contraction with the vierbein also acquires a fixed vev. The spacetime vev b_μ breaks diffeomorphisms. The converse is also true. If a nonscalar tensor vev on the spacetime manifold breaks diffeomorphisms, then the associated local tensor will have a vev that breaks local Lorentz symmetry. In the case of a scalar, the derivatives of the field will have vevs that break local Lorentz symmetry.

Next, the question of how many NG modes there are and where they reside was examined. Since there are six Lorentz symmetries and four diffeomorphisms, which can all be broken when a tensor with a sufficient number of indices acquires a fixed vev, this means that in general up to ten NG modes can appear. A general argument shows that these ten modes can all be absorbed as additional degrees of freedom in the vierbein. A simple counting argument supports this as well. The vierbein has 16 components. With Lorentz symmetry, six of these modes can be gauged away. They are usually chosen as the antisymmetric components. Similarly, diffeomorphisms can be used to remove four additional degrees of freedom. This leaves six vierbein modes in the general case. Einstein's theory has four of these modes as auxiliary, resulting in only two massless modes for the graviton. However, in a more general gravitational theory, there can be up to six propagating modes, which in a vierbein formalism are the six vierbein degrees of freedom. If Lorentz symmetry and diffeomorphisms are broken, then the ability to gauge away some of the vierbein degrees of freedom is lost. In particular, since up to ten symmetries can be broken, up to ten

additional modes (the NG modes) can appear in the vierbein.

The number of NG modes is also affected by the nature of the vev and by the fact that the symmetry is a spacetime symmetry. For example, in the case of a vector vev, which breaks three Lorentz symmetries and one diffeomorphism, it might be expected that there would be three massless NG Lorentz modes and one massless NG diffeomorphism mode. However, in the case where the vector vev is a constant, the diffeomorphism mode is found to be an auxiliary mode. It is also found that there are only two propagating massless Lorentz modes. The third Lorentz mode is found to be auxiliary as well. In this case, since the NG modes carry vector indices, it makes sense that a massless vector would only have two propagating modes. This clearly provides an example where the usual counting of NG modes (one massless mode per broken generator) does not hold for the case of a broken spacetime symmetry [33].

It was also found that the fate of the NG modes depends on the geometry. In Riemann or Minkowski spacetime, where the torsion is zero, the NG modes appear as additional massless or auxiliary modes in the vierbein. However, in Riemann-Cartan spacetime, which has nonzero torsion and where the spin connection has degrees of freedom that are independent from the vierbein, the possibility of a Higgs mechanism occurs. This is because a mass term for the spin connection can form when local Lorentz symmetry is spontaneously broken. If the theory permits massless propagating modes for the spin connection, then these modes can acquire a mass. In principle, the mechanism is straightforward. However, finding a ghost-free unbroken model with a propagating spin connection that is compatible with the mass term is challenging.

A specific vector model with spontaneous Lorentz breaking, called a bumblebee model, has been used to illustrate the behavior of the NG modes. For simplicity, this overview will concentrate entirely on this example for the case of a constant vev. All of the general features described above will be applicable.

Bumblebee models in a gravitational theory were first looked at by Kostelecky and Samuel as a simple model for investigating the consequences of spontaneous Lorentz violation [11]. Their properties have been studied in a variety of contexts [34]. Much of the attention has focused on models with a timelike vev. It has been suggested that if a NG diffeomorphism mode propagates in this case, then it would have an unusual dispersion relation [35].

One especially noteworthy feature of the bumblebee model occurs in Minkowski and Riemann spacetime. It is found (in the linearized theory) that the massless NG Lorentz modes behave essentially as the photon in an axial gauge [10]. Connections between Lorentz breaking and gauge fixing have been noted previously, leading to the suggestion that the photon is comprised of NG modes due to spontaneous Lorentz breaking [36, 37].

However, the approach of the bumblebee model is different. It is not a $U(1)$ gauge theory, since it contains a potential V that is not $U(1)$ invariant. The Lorentz breaking is therefore not a $U(1)$ gauge fixing choice. Nonetheless, the NG modes appear to behave at lowest order as photons in an axial gauge. Moreover, there are additional tell-tale signs of Lorentz breaking [10]. These include additional SME couplings in Riemann and Minkowski spacetime as well as anomalous gravitational couplings in the case of a Riemann geometry. This offers the possibility of letting experiments determine whether massless photons are the result of unbroken gauge symmetry or whether they might be due to spontaneously broken Lorentz symmetry.

4.1 Bumblebee Models

The definition of a bumblebee model is that it is a vector theory in which the vector field B^μ acquires a nonzero vev, which spontaneously breaks Lorentz symmetry. The lagrangian consists of a kinetic term for B^μ and a potential V that induces spontaneous Lorentz breaking. The potential is not $U(1)$ gauge invariant. Typically, the potential imposes a vev $b_a \neq 0$ for the vector in a local frame. The vierbein relates this back to the spacetime vector as $B_\mu = e_\mu{}^a b_a$. For simplicity, we assume a perturbative solution about a Minkowski background. This permits us to drop the distinction between latin and Greek indices and to write

$$e_{\mu\nu} = \eta_{\mu\nu} + (\tfrac{1}{2} h_{\mu\nu} + \chi_{\mu\nu}), \tag{29}$$

where the ten symmetric excitations $h_{\mu\nu} = h_{\nu\mu}$ are associated with the metric $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, while the six antisymmetric components $\chi_{\mu\nu} = -\chi_{\nu\mu}$ are the local Lorentz degrees of freedom. In this background, the vacuum solution takes the form

$$\langle B^\mu \rangle = b^\mu, \qquad \langle e_{\mu\nu} \rangle = \eta_{\mu\nu} \quad . \tag{30}$$

There are a number of choices for the kinetic and potential terms. Vector-current interactions and additional vector-curvature couplings that are forbidden in $U(1)$ gauge theory can be included as well [4, 9].

Here, as an illustrative example, we examine the model given by lagrangian

$$\begin{aligned} \mathcal{L}_B \quad = \quad & \frac{1}{2\kappa}(eR + \xi e B^\mu B^\nu R_{\mu\nu}) - \tfrac{1}{4} e B_{\mu\nu} B^{\mu\nu} \\ & -e\lambda(B_\mu B^\mu \pm b^2) - e B_\mu J^\mu, \end{aligned} \tag{31}$$

where $\kappa = 8\pi G$ and ξ is a coupling coefficient between the vector field and the curvature. The kinetic terms in this example are analogous to those in Einstein-Maxwell theory. However, in the general case in a Riemann-Cartan

spacetime, the torsion contributes to these terms and the field strength is defined by

$$B_{\mu\nu} = D_\mu B_\nu - D_\nu B_\mu, \tag{32}$$

where D_μ are covariant derivatives. The potential term is

$$V(B_\mu B^\mu \pm b^2) = \lambda(B_\mu B^\mu \pm b^2), \tag{33}$$

where λ is a Lagrange-multiplier field. It imposes the constraint that the vector field has a vev b^a obeying $b_a b^a = \mp b^2$ (with the sign corresponding to whether the vector is timelike or spacelike). The vector field can then be written in terms of the vierbein and can be expanded perturbatively to give

$$B^\mu = e^\mu{}_a b^a \approx b^\mu + (-\tfrac{1}{2}h^{\mu\nu} + \chi^{\mu\nu})b_\nu. \tag{34}$$

The vierbein degrees of freedom include the NG modes.

This model can be studied in a linearized approximation. The symmetric and antisymmetric components of the vierbein transform as

$$\begin{aligned} h_{\mu\nu} &\rightarrow h_{\mu\nu}, \\ \chi_{\mu\nu} &\rightarrow \chi_{\mu\nu} - \epsilon_{\mu\nu}, \end{aligned} \tag{35}$$

under infinitesimal Lorentz transformations, while under infinitesimal diffeomorphisms

$$\begin{aligned} h_{\mu\nu} &\rightarrow h_{\mu\nu} - \partial_\mu \xi_\nu - \partial_\nu \xi_\mu, \\ \chi_{\mu\nu} &\rightarrow \chi_{\mu\nu} - \tfrac{1}{2}(\partial_\mu \xi_\nu - \partial_\nu \xi_\mu). \end{aligned} \tag{36}$$

In these expressions, quantities of order (ϵh), $(\epsilon\chi)$, (ξh), $(\xi\chi)$, etc. are assumed small and hence negligible in the linearized treatment.

The NG modes can be found as the virtual fluctuations about the vacuum solution. These can be written as

$$\delta B^\mu = (B^\mu - b^\mu) \approx (-\tfrac{1}{2}h^{\mu\nu} + \chi^{\mu\nu})b_\nu. \tag{37}$$

It is useful to introduce projections on the transverse and longitudinal components of δB^μ along b^μ. Assuming $b^2 \neq 0$, these are given by

$$(P_\parallel)^\mu{}_\nu = \frac{b^\mu b_\nu}{b^\sigma b_\sigma}, \quad (P_\perp)^\mu{}_\nu = \delta^\mu{}_\nu - (P_\parallel)^\mu{}_\nu. \tag{38}$$

Defining the projected fluctuations as

$$\mathcal{E}^\mu = (P_\perp)^\mu{}_\nu \delta B^\nu, \qquad \rho^\mu = (P_\parallel)^\mu{}_\nu \delta B^\nu \approx b^\mu \rho, \tag{39}$$

where

$$\rho = -\frac{b^\mu h_{\mu\nu} b^\nu}{2 b^\sigma b_\sigma}. \tag{40}$$

lets us write the field B^μ as

$$B^\mu \approx (1+\rho)b^\mu + \mathcal{E}^\mu. \tag{41}$$

In terms of these projections, the NG Lorentz and diffeomorphism modes can be identified. Under a virtual local particle Lorentz transformation only components $\mathcal{E}^\mu$ obeying $b_\mu \mathcal{E}^\mu = 0$ are excited. These are the NG Lorentz modes, which evidently obey a condition similar to an axial-gauge condition in $U(1)$ gauge theory. If instead a virtual infinitesimal diffeomorphism is performed, only the longitudinal component ρ is excited. It can therefore be identified as the NG diffeomorphism mode. Note that a metric fluctuation about the vacuum solution,

$$\eta_{\mu\nu} \to g_{\mu\nu} \approx \eta_{\mu\nu} - \partial_\mu \xi_\nu - \partial_\nu \xi_\mu, \tag{42}$$

is generated by the diffeomorphism as well.

The dynamics of the NG modes depend on the background geometry. Three cases corresponding to Minkowski, Riemann, and Riemann-Cartan spacetime are examined in the following sections.

4.2 Minkowski Spacetime

In Minkowski spacetime, the curvature and torsion equal zero, and the metric can be written as

$$g_{\mu\nu} = \eta_{\mu\nu}. \tag{43}$$

The bumblebee lagrangian in Eq. (31) reduces to

$$\mathcal{L}_B = -\tfrac{1}{4} B_{\mu\nu} B^{\mu\nu} - \lambda(B_\mu B^\mu \pm b^2) - B_\mu J^\mu. \tag{44}$$

In this case, it is found that the diffeomorpism mode ρ cancels in $B_{\mu\nu}$. It is therefore an auxiliary mode and does not propagate. The Lorentz modes are contained in the projection $\mathcal{E}_\mu$. Renaming this as $\mathcal{E}_\mu \equiv A_\mu$ and calling the field strength $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ lets us rewrite the lagrangian as

$$\mathcal{L}_B \to \mathcal{L}_{\rm NG} \approx -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} - A_\mu J^\mu - b_\mu J^\mu + b^\mu \partial_\nu \Xi_\mu J^\nu, \tag{45}$$

where Ξ_μ is the longitudinal diffeomorphism mode ξ_μ promoted to an NG field. It is defined by $\rho = \partial_\mu \Xi^\mu$. Note that varying with respoct to this auxiliary mode yields the current-conservation law, $\partial_\mu J^\mu = 0$.

The lagrangian $\mathcal{L}_{\rm NG}$ is the effective quadratic lagrangian that governs the propagation of the NG modes in Minkowski space. The field A^μ has three degrees of freedom and automatically obeys an axial-gauge condition $b_\mu A^\mu = 0$. It contains the three Lorentz NG modes. Depending on the vev b_μ, the special cases of temporal gauge ($A^0 = 0$) and pure axial gauge ($A^3 = 0$) are possible.

It can be seen that in Minkowski spacetime the NG modes resemble those of a massless photon in $U(1)$ gauge theory in an axial gauge. Unlike the gauge theory case, however, where the masslessness of the photon is due to unbroken gauge symmetry, in this case the masslessness of the photon is a consequence of spontaneously broken Lorentz symmetry. An important question is whether this interpretation of the photon has experimentally verifiable consequences. Clearly, there is one additional interaction that does not hold for the usual photon in gauge theory. This is the Lorentz-violating term $b_\mu J^\mu$, where J^μ is the charge current. This term can be identified with the SME term with coefficient a^e_μ that occurs in the QED limit of the SME [8]. This type of SME coefficient if it is constant is known to be unobservable in experiments restricted to the electron sector [9, 8, 25]. However, it can generate signals in the quark and neutrino sectors. Thus, in experiments with multiple particle sectors, the idea that the photon results from spontaneous Lorentz violation can potentially be tested in Minkowski space.

4.3 Riemann Spacetime

In Riemann geometry in a vierbein formalism, the spin connection ${\omega_\mu}^{ab}$ appears in covariant derivatives. However, the metric requirement,

$$D_\lambda {e_\mu}^a = 0, \tag{46}$$

and the fact that the torsion vanishes permits the spin connection to be completely determined in terms of the vierbein as

$$\begin{aligned} {\omega_\mu}^{ab} &= \tfrac{1}{2} e^{\nu a}(\partial_\mu {e_\nu}^b - \partial_\nu {e_\mu}^b) - \tfrac{1}{2} e^{\nu b}(\partial_\mu {e_\nu}^a - \partial_\nu {e_\mu}^a) \\ &\quad - \tfrac{1}{2} e^{\alpha a} e^{\beta b} {e_\mu}^c (\partial_\alpha e_{\beta c} - \partial_\beta e_{\alpha c}). \end{aligned} \tag{47}$$

The spin connection has no independent degrees of freedom in Riemann spacetime, and the NG modes are still contained in the vierbein. In this case (with gravity), up to six of the 16 components of the vierbein can represent dynamical degrees of freedom associated with the gravitational fields.

We again consider the bumblebee lagrangian and vacuum as given in Eqs. (31) and (30), respectively. The projector-operator decomposition of B^μ reveals that there are four potential NG modes contained in $\mathcal{E}^\mu$ and ρ, and the axial-gauge condition $b_\mu \mathcal{E}^\mu = 0$ still holds in Riemann spacetime. The field strength $B_{\mu\nu}$ can be rewritten as

$$B_{\mu\nu} = (\partial_\mu {e_\nu}^a - \partial_\nu {e_\mu}^a) b_a, \tag{48}$$

which suggests that the propagation of the vierbein is modified by the bumblebee kinetic term.

The effective lagrangian for the NG modes can be found by expanding the bumblebee lagrangian to quadratic order, keeping couplings to matter currents and curvature. The result in terms of the decomposed fields is

$$\begin{aligned}
\mathcal{L}_{\rm NG} \;\; \approx \;\; & \frac{1}{2\kappa}[eR + \xi e b^\mu b^\nu R_{\mu\nu} + \xi e A^\mu A^\nu R_{\mu\nu} \\
& + \xi e \rho(\rho+2) b^\mu b^\nu R_{\mu\nu} + 2\xi e(\rho+1) b^\mu A^\nu R_{\mu\nu}] \\
& - \tfrac{1}{4} e F_{\mu\nu} F^{\mu\nu} - e A_\mu J^\mu - e b_\mu J^\mu + e b^\mu \partial_\nu \Xi_\mu J^\nu ,
\end{aligned} \tag{49}$$

where again the Lorentz modes are relabeled as $A_\mu \equiv \mathcal{E}_\mu$, which obeys $b_\mu A^\mu = 0$, and the field strength is $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$. The gravitational excitations $h_{\mu\nu}$ obey the condition $h_{\mu\nu} b^\mu = 0$.

The form of this effective lagrangian reveals that only two of the four potential NG modes propagate. These are the transverse Lorentz NG modes. The longitudinal Lorentz and the diffeomorphism NG modes are auxiliary. In particular, the curvature terms do not provide kinetic terms for ρ. This is because, metric fluctuations in the form of a diffeomorphism excitation produce only a vanishing contribution to the curvature tensor at linear order.

In Riemann spacetime, the NG Lorentz modes again resemble the photon in an axial gauge. The interaction with the charged current J_μ also has the appropriate form. However, possible signals for testing the idea that the photon is due to Lorentz violation can be found. In particular, there are unconventional couplings of the curvature with A^μ, ρ, and b^μ. The curvature couplings $eA^\mu A^\nu R_{\mu\nu}$, are forbidden by gauge invariance in conventional Einstein-Maxwell electrodynamics, but they can appear here in a theory with Lorentz violation. The term $\xi e b^\mu b^\nu R_{\mu\nu}/2\kappa$ corresponds to an SME coefficient of the $s^{\mu\nu}$ type in the gravity sector of the SME. The remaining terms also represent Lorentz-violating couplings that are included in the SME. Any of these signals could serve to provide experimental evidence for the idea that the photon is an NG mode due to spontaneous Lorentz violation.

4.4 Riemann-Cartan Spacetime

In a Riemann-Cartan spacetime, the vierbein ${e_\mu}^a$ and the spin connection ${\omega_\mu}^{ab}$ are independent degrees of freedom. As a result, the effects of spontaneous Lorentz breaking are very different from the cases of Minkowski and Riemann spacetime. In particular, it has been found that when the torsion is nonzero it is possible for a Higgs mechanism to occur [10]. This will be illustrated below in the context of the bumblebee model in Riemann-Cartan spacetime.

One immediate question concerning the possibility of a Higgs mechanism in a gravitational theory is whether the graviton acquires a mass or not.

Indeed, even a small mass for the graviton can modify the predictions of general relativity leading to disagreement with experiment [38]. However, it was shown some time ago that a conventional Higgs mechanism cannot give rise to a mass for the graviton since the terms that are generated involve derivatives of the metric [11].

A generic lagrangian for a theory with spontaneous Lorentz violation in Riemann-Cartan spacetime can be written as

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\rm SSB}. \tag{50}$$

Here, we assume $\mathcal{L}_0$ contains only gravitational terms formed from the curvature and torsion and describes the unbroken theory, while $\mathcal{L}_{\rm SSB}$ induces spontaneous Lorentz violation. For a Higgs mechanism to occur involving the spin connection, $\mathcal{L}_0$ should describe massless propagating modes for the spin connection prior to the spontaneous breaking of Lorentz symmetry. The theory should also be free of ghosts. It turns out that these conditions severely restrict the possibilities for model building. The number of ghost-free theories with massive and massless propagating spin connection modes is limited [39, 40]. The number of propagating modes in these models depends on the presence of additional accidental symmetries. The symmetry-breaking lagrangian $\mathcal{L}_{\rm SSB}$ typically breaks one or more of the accidental symmetries when the tensor field acquires a vev, which complicates the analysis of potential models.

In the bumblebee model in Eq. (31) the symmetry-breaking part of the lagrangian is

$$\mathcal{L}_{\rm SSB} = -\tfrac{1}{4} e B_{\mu\nu} B^{\mu\nu} - e\lambda (B_\mu B^\mu \pm b^2). \tag{51}$$

In a Riemann-Cartan spacetime, the field strength $B_{\mu\nu}$ is defined in Eq. (32). In terms of the vierbein and spin connection, it becomes

$$B_{\mu\nu} = ({e_\mu}^b {\omega_\nu}^a{}_b - {e_\nu}^b {\omega_\mu}^a{}_b) b_a. \tag{52}$$

Note that this expression reduces back to Eq. (48) in the limits of Riemann and Minkowski spacetimes, where the spin connection is given by Eq. (47).

When $B_{\mu\nu}$ is squared, quadratic terms in ${\omega_\mu}^a{}_b$ appear in the lagrangian, which perturbatively have the form

$$-\tfrac{1}{4} e B_{\mu\nu} B^{\mu\nu} \approx -\tfrac{1}{4} (\omega_{\mu\rho\nu} - \omega_{\nu\rho\mu})(\omega^{\mu\sigma\nu} - \omega^{\nu\sigma\mu}) b^\rho b_\sigma. \tag{53}$$

It is these quadratic terms that suggest that a Higgs mechanism can occur involving the absorption of the NG modes by the spin connection. It should be noted that this is only possible in Riemann-Cartan spacetime with nonzero torsion, since otherwise (as in Riemann spacetime) the spin connection has no independent degrees of freedom.

In Ref. [10], a number of different models for the kinetic terms $\mathcal{L}_0$ were considered. As mentioned, the difficulty in building a viable model with a

Higgs mechanism comes from finding a kinetic term describing propagating modes that are compatible with Eq. (53) as a mass term. If ghosts are permitted, this is straightforward. For example, with the choice

$$\mathcal{L}_0 = \tfrac{1}{4} R_{\lambda\kappa\mu\nu} R^{\lambda\kappa\mu\nu}. \tag{54}$$

all the fields $\omega_{\lambda\mu\nu}$ with $\lambda \neq 0$ propagate as massless modes. When this is combined with $\mathcal{L}_{\rm SSB}$, we find that among the propagating modes in the linearized theory there is a massive mode. Other examples can be studied as well and are aided by decomposing the fields $\omega_{\lambda\mu\nu}$ according to their spin-parity projections J^P in three-dimensional space. This reveals that the mass term consists of a physical 1^+ mode and a 1^- gauge mode. Models can be found in which $\mathcal{L}_0$ includes a massless 1^+ mode. However, typically the propagating massless modes involve combinations of J^P projections, which makes finding compatibility with $\mathcal{L}_{\rm SSB}$ all the more challenging.

In the end, a number of issues remain open for future investigation. Studies of the large variety of possible Lorentz-invariant lagrangians $\mathcal{L}_0$ can lead to new models in which the spin connection acquires a mass due to spontaneous Lorentz breaking. Different choices for $\mathcal{L}_{\rm SSB}$ can also be considered, including ones in which the spontaneous Lorentz violation involves one or more tensor fields. This would certainly affect the dynamics of the NG modes as well. From a broader theoretical point of view, the incorporation of spontaneous Lorentz violation in theories with torsion opens up a new arena in the search for ghost-free models with propagating massive modes.

Certainly, there are implications for phenomenology in the context of Riemann-Cartan spacetime. The relevant mass scale in the Higgs mechanism is set by b^2. Even if this is on the order of the Planck mass, the existence of fields associated with Lorentz violation could have effects on cosmology, black holes, and gravitational radiation. Since all of the relevant terms in any of these models are included in the SME in Riemann-Cartan spacetime, a systematic approach would be to investigate possible new signals in that context.

5 Phenomenology

The minimal SME described in Section (3.1) has been used extensively in recent years by experimentalists and theorists to search for leading-order signals of Lorentz violation. To date, Planck-scale sensitivity has been attained to the dominant SME coefficients in a number of experiments involving different particle sectors. These include experiments with photons [29, 41, 42, 43, 44, 45, 46], electrons [47, 48, 49, 50, 51, 52, 53], protons and neutrons [54, 55, 56, 57, 58, 59], mesons [60, 61], muons [62, 63, 64], neutrinos [8, 17, 65, 66, 67], and the Higgs [68]. It should be noted that

despite the length of this list of experiments, a substantial portion of the SME coefficient space remains unexplored.

In the remaining sections, an overview of some of the recent tests of Lorentz and CPT symmetry in a Minkowski background will be given. In particular, since many of the sharpest test are performed in high-precision atomic and particle experiments involving photons and charged particles, much of the focus will be on the QED limit of the minimal SME. However, two other particle sectors are briefly described as well. These involve testing Lorentz and CPT symmetry with mesons and neutrinos.

5.1 Mesons

Experiments with mesons have long provided some of the sharpest tests of CPT. Since CPT and Lorentz symmetry are intertwined in field theory, these experiments also provide additional tests of Lorentz symmetry. Investigations in the context of the SME have found very high sensitivity to the CPT-odd a_μ coefficients in the SME.

The time evolution of a meson P^0 and its antimeson $\overline{P^0}$ is governed by a 2×2 effective hamiltonian Λ in a description based on the Schrödinger equation. Here, P represents one of the neutral mesons K, D, B_d, B_s. The hamiltonian can be written as [61, 69]

$$\Lambda = \tfrac{1}{2}\Delta\lambda \begin{pmatrix} U+\xi & VW^{-1} \\ VW & U-\xi \end{pmatrix} , \tag{55}$$

where the parameters $UVW\xi$ are complex. The factor $\Delta\lambda/2$ ensures these parameters are dimensionless. Imposing conditions on the trace and determinant gives the relations $U \equiv \lambda/\Delta\lambda$ and $V \equiv \sqrt{1-\xi^2}$. The independent complex parameters $W = w\exp(i\omega)$ and $\xi = \mathrm{Re}[\xi] + i\mathrm{Im}[\xi]$ have four real components. One is physically unobservable. The argument ω changes under a phase redefinition of the P^0 wave function. The three others are physical. The two real numbers $\mathrm{Re}[\xi]$, $\mathrm{Im}[\xi]$ determine the amount of CPT violation, with CPT preserved if and only if both are zero.

The dominant CPT-violating contributions to the effective hamiltonian Λ can be calculated as expectation values of interaction terms in the SME. The result in terms of ξ is

$$\xi \sim \beta^\mu \Delta a_\mu \quad , \tag{56}$$

where $\beta^\mu = \gamma(1, \vec{\beta})$ is the four-velocity of the P meson in the laboratory frame and the coefficients Δa_μ are combinations of SME coefficients.

The 4-velocity (and 4-momentum) dependence in Eq. (56) shows explicitly that CPT violation cannot be described with a constant complex parameter in quantum field theory [61]. Nonetheless, most experiments have

fit their data to a constant value of ξ. Experiments in the kaon system [70], for example, have attained bounds of order 10^{-4} on the real and imaginary parts of ξ. More recently, however, analyses have been performed taking into account that in an experiment Δa_μ varies with the magnitude and direction of the momentum and with sidereal time as the Earth rotates. These experiments have attained sensitivities to Δa_μ on the order of 10^{-20} GeV in the kaon system and 10^{-15} GeV in the D system [60]. Additional bounds for the B_d and B_s systems can be obtained as well in future analyses.

5.2 Neutrinos

A general analysis in the context of the SME has searched for possible signals of Lorentz violation in neutrino physics [66]. Among other things, it looked at how free neutrinos with Dirac and Majorana couplings oscillate in the presence of Lorentz violation. Remarkably, a number of possible models exist in which Lorentz violation (with or without massive neutrinos) contributes to neutrino oscillations. One two-parameter model in particular, consisting of massless neutrinos, called the bicycle model, reproduces features in observed data (except for the LSND experiment). Indeed, a statistical analysis performed using data from Super-Kamiokande on atmospheric neutrinos finds that the fit based on the bicycle model is essentially as good (within a small marginal error) to the fit based on small mass differences [65]. Further investigations looking for sidereal time variations will be able to distinguish oscillations associated with Lorentz violation from those due to small mass differences.

5.3 QED Sector

Traditionally, many of the sharpest tests of Lorentz and CPT symmetry have been made with photons or in particle or atomic systems where the predominant interactions are described by QED. This would include the original Michelson-Morley experiments and their modern-day versions [42, 43, 44]. The Lorentz tests known as Hughes-Drever experiments are atomic experiments in which two high-precision atomic clocks consisting of different atomic species are compared as the Earth rotates [54]. These provide exceptionally sharp tests of Lorentz symmetry. Similarly, some of the best CPT tests for leptons and baryons – involving direct comparisons of particles and antiparticles – are made by atomic physicists working with Penning traps [47, 48, 59].

In order to look for the leading order signals of Lorentz and CPT violation in these types of experiments, it is useful to work with a subset of the minimal SME lagrangian that is relevant to experiments in QED systems. The QED limit of the minimal SME can be written as

$$\mathcal{L}_{\rm QED} = \mathcal{L}_0 + \mathcal{L}_{\rm int} \quad . \tag{57}$$

The lagrangian $\mathcal{L}_0$ contains the usual Lorentz-invariant terms in QED describing photons, massive charged fermions, and their conventional couplings, while $\mathcal{L}_{\rm int}$ contains the Lorentz-violating interactions. Since the minimal SME in flat spacetime is restricted to the remormalizable and gauge-invariant terms in the full SME, the QED sector interactions in $\mathcal{L}_{\rm int}$ have a finite number of terms. For the case of photons and a single fermion species ψ the Lorentz-violating terms are given by [71]

$$\begin{aligned}\mathcal{L}_{\rm int} \quad = \quad & -a_\mu \bar{\psi}\gamma^\mu\psi - b_\mu \bar{\psi}\gamma_5\gamma^\mu\psi + ic_{\mu\nu}\bar{\psi}\gamma^\mu D^\nu\psi \\ & \quad +id_{\mu\nu}\bar{\psi}\gamma_5\gamma^\mu D^\nu\psi - \tfrac{1}{2}H_{\mu\nu}\bar{\psi}\sigma^{\mu\nu}\psi \\ & -\tfrac{1}{4}(k_F)_{\kappa\lambda\mu\nu}F^{\kappa\lambda}F^{\mu\nu} + \tfrac{1}{2}(k_{AF})^\kappa \epsilon_{\kappa\lambda\mu\nu}A^\lambda F^{\mu\nu} \quad . \end{aligned} \tag{58}$$

Here, $iD_\mu \equiv i\partial_\mu - qA_\mu$. The terms with coefficients a_μ, b_μ and $(k_{AF})_\mu$ are odd under CPT, while those with $H_{\mu\nu}$, $c_{\mu\nu}$, $d_{\mu\nu}$, and $(k_F)_{\kappa\lambda\mu\nu}$ preserve CPT. All seven terms break Lorentz symmetry. In general, superscript labels will be added to these parameters to denote the particle species.

This lagrangian emerges naturally from the minimal SME sector for charged leptons, following the usual assumptions of electroweak symmetry breaking and mass generation. Lagrangian terms of the same form are expected to describe protons and neutrons in QED systems as well, but where the SME coefficients represent composites stemming from quark and gluon interactions. It is certainly the case that QED and its relativistic quantum-mechanical limits describe proton and neutron electromagnetic interactions in atoms in excellent agreement with experiments. Defining terms involving composite SME parameters for protons and neutrons is therefore a reasonable extension of the theory. The QED extension of the SME treats protons and neutrons as the basic constituents of the theory. The lagrangian $\mathcal{L}_{\rm int}$ then contains the most general set of Lorentz-violating interactions in this context.

Since the corrections due to Lorentz violation at low energy are known to be small, it is sufficient in many situations to work in the context of relativistic quantum mechanics using perturbation theory. To do so, a Hamiltonian is needed such that

$$i\partial_0\chi = \hat{H}\chi \quad , \tag{59}$$

where $\hat{H} = \hat{H}_0 + \hat{H}_{\rm pert}$. The perturbative hamiltonian $\hat{H}_{\rm pert}$ associated with Lorentz violation can be generated using a Foldy-Wouthuysen approach and by making appropriate field redefinitions [49, 57]. The result for a massive fermion particle is

$$\begin{aligned}\hat{H}_{\rm pert} \quad = \quad & a_\mu\gamma^0\gamma^\mu - b_\mu\gamma_5\gamma^0\gamma^\mu - c_{00}m\gamma^0 - i(c_{0j}+c_{j0})D^j \\ & +i(c_{00}D_j - c_{jk}D^k)\gamma^0\gamma^j - d_{j0}m\gamma_5\gamma^j + i(d_{0j}+d_{j0})D^j\gamma_5 \\ & +i(d_{00}D_j - d_{jk}D^k)\gamma^0\gamma_5\gamma^j + \tfrac{1}{2}H_{\mu\nu}\gamma^0\sigma^{\mu\nu} \quad . \end{aligned} \tag{60}$$

Here, the letters j, k, l, etc. represent the three spatial directions in a laboratory frame. The $j = 3$ (or z direction) is usually chosen as the quantization

axis. The corresponding hamiltonian for the antiparticle can be obtained using charge conjugation.

The SME coefficients are expected to be fixed with respect to a nonrotating coordinate frame. As a result, the SME coefficients b_0, b_j, etc. would change as the Earth moves. In order to give measured bounds in a consistent manner, a nonrotating frame is chosen. Often, this is chosen as a sun-centered frame using celestial equatorial coordinates. These are denoted using upper-case letters T,X,Y,Z. Typically, experiments sensitive to sidereal time variations are sensitive to a combination of parameters, which are denoted using tildes. For example, the b_μ tilde coefficients with $\mu = j$ are defined as

$$\tilde{b}_j^e \equiv b_j^e - m d_{j0}^e - \tfrac{1}{2}\varepsilon_{jkl} H_{kl}^e \quad , \tag{61}$$

These combinations are projected onto the nonrotating frame, where the components with respect to the celestial equatorial coordinate frame are b_X^e, b_Y^e, b_Z^e, etc. The relation between the laboratory and nonrotating components is

$$\begin{aligned}
\tilde{b}_1^e &= \tilde{b}_X^e \cos\chi \cos\Omega t + \tilde{b}_Y^e \cos\chi \sin\Omega t - \tilde{b}_Z^e \sin\chi, \\
\tilde{b}_2^e &= -\tilde{b}_X^e \sin\Omega t + \tilde{b}_Y^e \cos\Omega t, \\
\tilde{b}_3^e &= \tilde{b}_X^e \sin\chi \cos\Omega t + \tilde{b}_Y^e \sin\chi \sin\Omega t + \tilde{b}_Z^e \cos\chi.
\end{aligned} \tag{62}$$

The angle χ is between the $j = 3$ lab axis and the direction of the Earth's rotation axis, which points along Z. The angular frequency $\Omega \simeq 2\pi/(23\text{h}\,56\text{m})$ is that corresponding to a sidereal day.

6 Tests in QED

Before examining individual tests of Lorentz symmetry in QED systems, it is useful to examine some of the more general results that have emerged from these investigations. One general feature is that sensitivity to Lorentz and CPT violation in these experiments stems primarily from their ability to detect very small anomalous energy shifts. While many of the experiments were originally designed to measure specific quantities, such as charge-to-mass ratios of particles and antiparticles or differences in g factors, it is now recognized that these experiments are most effective as Lorentz and CPT tests when all of the energy levels in the system are investigated for possible anomalous shifts. As a result of this, a number of new signatures of Lorentz and CPT violation have been discovered in recent years that were overlooked previously.

A second general feature concerns how these atomic experiments are typically divided into two groups. The first (Lorentz tests) looks for sidereal time variations in the energy levels of a particle or atom. The second (CPT tests) looks for a difference in the energy levels between a particle (or

atom) and its antiparticle (or antiatom). What has been found is that the sensitivity to Lorentz and CPT violation in these two classes of experiments is not distinct. Experiments traditionally viewed as Lorentz tests are also sensitive to CPT symmetry and vice versa. Nonetheless, it is important to keep in mind that that there are differences as well. For example, the CPT experiments comparing matter and antimatter are directly sensitive to CPT-violating parameters, such as b_μ, whereas Lorentz tests are sensitive to combinations of CPT-preserving and CPT-violating parameters, which are denoted using a tilde. Ultimately, both clases of experiments are important and should be viewed as complementary.

It has become common practice to express sensitivities to Lorentz and CPT violation in terms of the SME coefficients. This provides a straightforward approach that allows comparisons across different types of experiments. Since each different particle sector in the QED extension has an independent set of Lorentz-violating SME coefficients, these are distinguished using superscript labels. A thorough investigation of Lorentz and CPT violation necessarily requires looking at as many different particle sectors as possible.

6.1 Photons

The lagrangian describing a freely propagating photon in the presence of Lorentz violation is given by [45]

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \tfrac{1}{4}(k_F)_{\kappa\lambda\mu\nu}F^{\kappa\lambda}F^{\mu\nu} \ + \tfrac{1}{2}(k_{AF})^\kappa \epsilon_{\kappa\lambda\mu\nu}A^\lambda F^{\mu\nu}, \qquad (63)$$

where the field strength $F_{\mu\nu}$ is defined by $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$.

The coefficient k_{AF}, which is odd under CPT, has been investigated extensively both theoretically and experimentally [41, 45]. Theoretically, it is found that this term leads to negative-energy contributions and is a potential source of instability in the theory. One solution is to set k_{AF} to zero, which has been shown to be consistent with radiative corrections in the SME. However, stringent experimental constraints also exist consistent with $k_{AF} \approx 0$. These result from studying the polarization of radiation from distant radio galaxies. In what follows, we will therefore ignore the effects of the k_{AF} terms.

The terms with coefficients k_F, which is even under CPT, have been investigated more recently [45]. These terms provide positive-energy contributions. There are 19 independent components in the k_F coefficients. It is useful to rewrite them in terms of a new set, $\tilde{\kappa}_{e+}$, $\tilde{\kappa}_{e-}$, $\tilde{\kappa}_{o+}$, $\tilde{\kappa}_{o-}$, and $\tilde{\kappa}_{\rm tr}$. Here, $\tilde{\kappa}_{e+}$, $\tilde{\kappa}_{e-}$, and $\tilde{\kappa}_{o-}$ are 3×3 traceless symmetric matrices (with 5 independent components each), while $\tilde{\kappa}_{o+}$ is a 3×3 antisymmetric matrix (with 3 independent components), and the remaining coefficient $\tilde{\kappa}_{\rm tr}$ is the only rotationally invariant component.

The lagrangian can be written in terms of the new set and the usual

electric and magnetic fields $\vec{E}$ and $\vec{B}$ as follows:

$$\begin{aligned}\mathcal{L} &= \tfrac{1}{2}[(1+\tilde{\kappa}_{\rm tr})\vec{E}^2 - (1-\tilde{\kappa}_{\rm tr})\vec{B}^2] + \tfrac{1}{2}\vec{E}\cdot(\tilde{\kappa}_{e+}+\tilde{\kappa}_{e-})\cdot\vec{E} \\ &\quad -\tfrac{1}{2}\vec{B}\cdot(\tilde{\kappa}_{e+}-\tilde{\kappa}_{e-})\cdot\vec{B} + \vec{E}\cdot(\tilde{\kappa}_{o+}+\tilde{\kappa}_{o-})\cdot\vec{B} \quad . \qquad (64)\end{aligned}$$

This lagrangian gives rise to modifications of Maxwell's equations, which have been explored in recent astrophysical and laboratory experiments. Ten of the coefficients, $\tilde{\kappa}_{e+}$ and $\tilde{\kappa}_{o-}$, lead to birefrigence of light. Bounds on these parameters of order 2×10^{-32} have been obtained from spectropolarimetry of light from distant galaxies [45]. The nine coefficients, $\tilde{\kappa}_{\rm tr}$, $\tilde{\kappa}_{e-}$, and $\tilde{\kappa}_{o+}$, have been bounded in a series of recent laboratory photon experiments. Seven of the eight $\tilde{\kappa}_{e-}$ and $\tilde{\kappa}_{o+}$ coefficients, have been bounded in experiments using optical and microwave cavities. Sensitivities on the order of $\tilde{\kappa}_{o+} \lesssim 10^{-11}$ and $\tilde{\kappa}_{e-} \lesssim 10^{-15}$ have been attained [42]. The trace coefficient has been estimated to have an upper bound of $\tilde{\kappa}_{\rm tr} \lesssim 10^{-4}$ from Ives-Stilwell experiments [43]. The remaining $\tilde{\kappa}_{e-}$ coefficient has recently been bounded at the level of 10^{-14} using a rotating apparatus [44].

6.2 Penning Traps

There are primarily two leading-order signals of Lorentz and CPT violation that can be searched for in experiments in Penning traps [49]. One is a traditional CPT test, comparing particles and antiparticles, while the other is a Lorentz test that looks for sidereal time variations. Both types of signals have been investigated in recent years in experiments with electrons and positrons. The experiments involve making high-precision measurements of the anomaly frequency ω_a and the cyclotron frequency ω_c of the trapped electrons and/or positrons.

The first test was a reanalysis was performed by Dehmelt's group using existing data for electrons and positrons in a Penning trap [47]. The idea was to look for an instantaneous difference in the anomaly frequencies of electrons and positrons, which can be nonzero when CPT and Lorentz symmetry are broken. Dehmelt's original measurements of $g-2$ did not involve looking for possible instantaneous variations in ω_a. Instead, the ratio ω_a/ω_c was computed using averaged values. However, Lorentz-violating corrections to the anomaly frequency ω_a can occur even if the g factor remains unchanged. An alternative analysis therefore looks for an instantaneous difference in the electron and positron anomaly frequencies. The new bound found by Dehmelt's group can be expressed in terms of the parameter b_3^e, which is the component of b_μ^e along the quantization axis in the laboratory frame. The bound they obtained is $|b_3^e| \lesssim 3\times10^{-25}$ GeV.

The second signal for Lorentz and CPT violation in the electron sector involves measurements of the electron alone [48]. Here, the idea is that the Lorentz and CPT-violating interactions depend on the orientation of

the quantization axis in the laboratory frame, which changes as the Earth turns on its axis. As a result, both the cyclotron and anomaly frequencies have small corrections which cause them to exhibit sidereal time variations. Such a signal can be measured using just electrons, which eliminates the need for comparison with positrons. The bounds in this case are given with respect to a nonrotating coordinate system such as celestial equatorial coordinates. The interactions involve a combination of laboratory-frame components that couple to the electron spin. The combination is denoted as $\tilde{b}_3^e \equiv b_3^e - m d_{30}^e - H_{12}^e$. The bound can be expressed in terms of components X, Y, Z in the nonrotating frame. It is given as $|\tilde{b}_J^e| \lesssim 5 \times 10^{-25}$GeV for $J = X, Y$.

Although no $g-2$ experiments have been made for protons or antiprotons, there have been recent bounds obtained on Lorentz violation in comparisons of cyclotron frequencies of antiprotons and H^- ions confined in a Penning trap [59]. In this case the sensitivity is to the dimensionless parameters $c_{\mu\nu}^p$. Future experiments with protons and antiprotons will be able to provide tests that are sensitive to b_μ^p

6.3 Clock-Comparison Experiments

The classic Hughes-Drever experiments are atomic clock-comparison tests of Lorentz invariance [54, 57]. There have been a number of different types of these experiments performed over the years, with steady improvements in their sensitivity. They involve making high-precision comparisons of atomic clock signals as the Earth rotates. The clock frequencies are typically hyperfine or Zeeman transitions. Many of the sharpest Lorentz bounds for the proton, neutron, and electron stem from atomic clock-comparison experiments. For example, Bear *et al.* in Ref. [54] used a two-species noble-gas maser to test for Lorentz and CPT violation in the neutron sector. They obtain a bound $|\tilde{b}_J^n| \lesssim 10^{-31}$GeV for $J = X, Y$, which is currently the best bound for the neutron sector.

It should also be pointed out that certain assumptions about the nuclear configurations must be made to obtain bounds in clock-comparison experiments. For this reason, these bounds should be viewed as good to within about an order of magnitude. To obtain cleaner bounds it is necessary to consider simpler atoms or to perform more sophisticated nuclear modeling.

Note as well that these Earth-based laboratory experiments are not sensitive to Lorentz-violation coefficients along the $J = Z$ direction parallel to Earth's rotation axis. They also neglect the velocity effects due to Earth's motion around the sun, which would lead to bounds on the timelike components along $J = T$. These limitations can be overcome by performing experiments in space or by using a rotation platform. The earth's motion can also be taken into account. A recent boosted-frame analysis of the dual noble-gas maser experiment has yielded bounds on the order of 10^{-27}

GeV on many boost-dependent SME coefficients for the neutron that were previously unbounded [56].

6.4 Experiments in Space

Clock-comparison experiments performed in space would have several advantages over traditional ground-based experiments [58]. For example, a clock-comparison experiment conducted aboard the International Space Station (ISS) would be in a laboratory frame that is both rotating and boosted. It would therefore immediately gain sensitivity to both the Z and timelike directions. This would more than triple the number of Lorentz-violation parameters that are accessible in a clock-comparison experiment. Another advantage of an experiment aboard the ISS is that the time needed to acquire data would be greatly reduced (by approximately a factor of 16). In addition, new types of signals would emerge that have no analogue in traditional Earth-based experiments. The combination of these advantages should result in substantially improved limits on Lorentz and CPT violation. Unfortunately, the USA has canceled its missions aimed at testing fundamental physics aboard the ISS. However, there is still a European mission planned for the ISS which will compare atomic clocks and H masers. Therefore, the opportunity to perform these new Lorentz and CPT tests is still a possibility.

6.5 Hydrogen and Antihydrogen

Hydrogen atoms have the simplest nuclear structure, and antihydrogen is the simplest antiatom. These atoms (or antiatoms) therefore provide opportunities for conducting especially clean Lorentz and CPT tests involving protons and electrons.

There are three experiments underway at CERN that can perform high-precision Lorentz and CPT tests in antihydrogen [12]. Two of the experiments (ATRAP and ATHENA) intend to make high-precision spectroscopic measurements of the 1S-2S transitions in hydrogen and antihydrogen. These are forbidden (two-photon) transitions that have a relative linewidth of approximately 10^{-15}. The ultimate goal is to measure the line center of this transition to a part in 10^3 yielding a frequency comparison between hydrogen and antihydrogen at a level of 10^{-18}. An analysis of the 1S-2S transition in the context of the SME shows that the magnetic field plays an important role in the attainable sensitivity to Lorentz and CPT violation [50]. For instance, in free hydrogen in the absence of a magnetic field, the 1S and 2S levels are shifted by equal amounts at leading order. As a result, in free H or $\bar{\mathrm{H}}$ there are no leading-order corrections to the 1S-2S transition frequency. In a magnetic trap, however, there are fields that can mix the spin states in the four different hyperfine levels. Since the Lorentz-violating interactions

depend on the spin orientation, there will be leading-order sensitivity to Lorentz and CPT violation in comparisons of 1S-2S transitions in trapped hydrogen and antihydrogen. At the same time, however, these transitions are field-dependent, which creates additional experimental challenges that would need to be overcome.

An alternative to 1S-2S transitions is to consider the sensitivity to Lorentz violation in ground-state Zeeman hyperfine transitions. It is found that there are leading-order corrections in these levels in both hydrogen and antihydrogen [50]. The ASACUSA group at CERN is planning to measure the Zeeman hyperfine transitions in antihydrogen. Such measurements will provide a direct CPT test.

Experiments with hydrogen alone have been performed using a maser [55]. They attain exceptionally sharp sensitivity to Lorentz and CPT violation in the electron and proton sectors of the SME. These experiments use a double-resonance technique that does not depend on there being a field-independent point for the transition. The sensitivity for the proton attained in these experiments is $|\tilde{b}_J^p| \lesssim 10^{-27}$ GeV. Due to the simplicity of hydrogen, this is an extremely clean bound and is currently the most stringent test of Lorentz and CPT violation for the proton.

6.6 Muon Experiments

Experiments with muons involve second-generation leptons and provide tests of CPT and Lorentz symmetry that are independent of the tests involving electrons. There are several different types of experiments with muons that have recently been conducted, including muonium experiments [62] and $g-2$ experiments with muons at Brookhaven [63]. In muonium, experiments measuring the frequencies of ground-state Zeeman hyperfine transitions in a strong magnetic field have the greatest sensitivity to Lorentz and CPT violation. A recent analysis has searched for sidereal time variations in these transitions. A bound at the level of $|\tilde{b}_J^\mu| \leq 2 \times 10^{-23}$ GeV has been obtained [62]. In relativistic $g-2$ experiments using positive muons with "magic" boost parameter $\delta = 29.3$, bounds on Lorentz-violation parameters are possible at a level of 10^{-25} GeV. However, the analysis of these experiments is still underway at Brookhaven.

6.7 Spin Polarized Torsion Pendulum

Experiments using spin polarized torsion pendula have been conducted at the University of Washington and in Taiwan. These experiments currently provide the sharpest bounds on Lorentz and CPT symmetry in the electron sector [52]. These experiments are able to achieve very high sensitivity to Lorentz violation because the torsion pendula have a huge number of aligned electron spins but a negligible magnetic field.

The pendulum at the University of Washington is built out of a stack of toroidal magnets, which in one version of the experiment achieved a net electron spin $S \simeq 8 \times 10^{22}$. The apparatus is suspended on a rotating turntable and the time variations of the twisting pendulum are measured. An analysis of this system shows that in addition to a signal having the period of the rotating turntable, the effects due to Lorentz and CPT violation also cause additional time variations with a sidereal period caused by the rotation of the Earth. The group at the University of Washington has analyzed data taken in 1998 and find that thay have sensitivity to the electron coefficients at the levels of $|\tilde{b}^e_J| \lesssim 10^{-29}$ GeV for $J = X, Y$ and $|\tilde{b}^e_Z| \lesssim 10^{-28}$ GeV. More recently, a new pendulum has been built, and it is expected that 20-fold improved sensitivities will be attained [72].

The Taiwan experiment also uses a rotating torsion pendulum, which is made of a ferrimagnetic material. This group achieved a net polarization of $S \simeq 8.95 \times 10^{22}$ electrons in their pendulum. The bounds they obtain for the electron are at the levels of $|\tilde{b}^e_J| \lesssim 3.1 \times 10^{-29}$ GeV for $J = X, Y$ and $|\tilde{b}^e_Z| \lesssim 7.1 \times 10^{-28}$ GeV.

7 Conclusions

This overview describes the development and use of the SME as the theoretical framework describing Lorentz violation in the context of field theory. The philosophy of the SME is that any interactions that are observer invariant and involve known fields at low energy are included in the theory. As an incremental first step, the minimal SME (and its QED limit) can be constructed. This theory maintains gauge invariance and power-counting renormalizability. It is the suitable framework for investigating leading-order signals of Lorentz violation.

In addition to constructing the SME, we have examined the special case of spontaneous Lorentz breaking. In particular, the question of what the fate of the Nambu-Goldstone modes is when Lorentz symmetry is spontaneously broken has been addressed. We have demonstrated that spontaneous particle Lorentz violation is accompanied by spontaneous particle diffeomorphism violation and vice versa, and that up to 10 NG modes can appear. These modes can comprise 10 of the 16 modes of the vierbein that in a Lorentz-invariant theory are gauge degrees of freedom. The fate of the NG modes is found to depend also on the spacetime geometry and on the behavior of the tensor vev inducing spontaneous Lorentz violation. These results have been illustrated using a bumblebee model. In Minkowski and Riemann spacetimes, it is found that the NG modes propagate like the photon in an axial gauge. In Riemann-Cartan spacetimes, the interesting possibility exists that the spin connection could absorb the propagating NG modes in a gravitational version of the Higgs mechanism. This unique feature of gravity

Expt	Sector	Params ($J = X, Y$)	Bound (GeV)
Penning Trap	electron	$\tilde{b}^e_J$	5×10^{-25}
Hg-Cs clock comparison	electron	$\tilde{b}^e_J$	$\sim 10^{-27}$
	proton	$\tilde{b}^p_J$	$\sim 10^{-27}$
	neutron	$\tilde{b}^n_J$	$\sim 10^{-30}$
He-Xe dual maser	neutron	$\tilde{b}^n_J$	$\sim 10^{-31}$
H maser	electron	$\tilde{b}^e_J$	10^{-27}
	proton	$\tilde{b}^p_J$	10^{-27}
Muonium	muon	$\tilde{b}^\mu_J$	2×10^{-23}
Spin Pendulum	electron	$\tilde{b}^e_J$	10^{-29}
		$\tilde{b}^e_Z$	10^{-28}

Table 1: Summary of leading-order bounds for the parameter $\tilde{b}_J$.

theories with torsion may offer another phenomenologically viable route for constructing realistic models with spontaneous Lorentz violation.

Phenomenology has been investigated using the minimal SME. Experiments in QED systems continue to provide many of the sharpest tests of Lorentz and CPT symmetry. In recent years, a number of new astrophysical and laboratory tests have been performed that have lead to substantially improved sensitivities for the photon. Similarly, atomic experimentalists continue to find ways of improving the sensitivity to Lorentz violation in many of the matter sectors of the SME. For comparison across different atomic experiments a summary of recent bounds on the $\tilde{b}_J$ coefficients in the minimal SME is given in Table 1. These bounds are within the range of sensitivity associated with suppression factors arising from the Planck scale. A more complete table would list all of the coefficients in the minimal SME. Note that many SME coefficients have still not been measured. Future experiments, in particular those performed in boosted frames, are likely to provide sensitivity to many of these currently unmeasured SME coefficients. In addition, the overall sensitivity of these experiments is expected to improve over the coming years.

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